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BOUNDS OF THE PERIMETER OF AN ELLIPSE USING ARITHMETIC, GEOMETRIC AND HARMONIC MEANS

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Abstract. In this paper, we present several bounds for the perimeter of an ellipse in terms of arithmetic, geometric, and harmonic means, which improve some known results.

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