ON NECESSARY AND SUFFICIENT CONDITIONS
FOR VARIABLE EXPONENT HARDY INEQUALITY

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Abstract. In this paper we derive close necessary and sufficient conditions on the regularity of the exponent functions p, β such that the variable exponent Hardy inequality holds

$$\left\| x^{\beta(x)-1} \int_0^x f(t)dt \right\|_{L^{p(.)}(0,l)} \leqslant C \left\| x^{\beta(x)} f \right\|_{L^{p(.)}(0,l)}$$

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