

## REVERSE ORDER LAW FOR WEIGHTED MOORE–PENROSE INVERSES OF MULTIPLE MATRIX PRODUCTS

ZHIPING XIONG AND YINGYING QIN

*Abstract.* In this paper by using some matrix rank theories, we derive equivalent conditions for reverse order law of weighted Moore-Penrose inverses of multiple matrix products. In addition, we also give a variety of necessary and sufficient conditions for the reverse product  $(A_n)_{M_n, M_{n+1}}^\dagger (A_{n-1})_{M_{n-1}, M_n}^\dagger \cdots (A_1)_{M_1, M_2}^\dagger$  to be a  $\{1\}$ -,  $\{1, 2\}$ -,  $\{1, 3M_1\}$ -,  $\{1, 4M_{n+1}\}$ -,  $\{1, 2, 3M_1\}$ - or  $\{1, 2, 4M_{n+1}\}$ -inverse of matrix product  $A_1 A_2 \cdots A_n$ .

*Mathematics subject classification (2010):* 15A03, 15A09, 15A24.

*Keywords and phrases:* Reverse order law, generalized inverse, weighted generalized inverse, elementary block matrix operations, matrix rank theory.

### REFERENCES

- [1] A. BEN-ISRAEL AND T. N. E. GREVILLE, *Generalized Inverses: Theory and Applications*, Wiley-Interscience, 1974, 2nd Edition, Springer-Verlag, New York, 2002.
- [2] D. T. BARWICK AND J. D. GILBERT, *On generalization of the reverse order law with related results*, SIAM J. Appl. Math. **27** (1974), 326–330.
- [3] D. S. DJORDJEVIĆ, *Furthuer results on the reverse order law for generalized inverses*, SIAM J. Matrix. Anal. Appl. **29** (2007), 1242–1246.
- [4] T. N. E. GREVILLE, *Note on the generalized inverse of a matrix product*, SIAM Review **8** (1966), 518–521.
- [5] R. E. HARTWIG, *Block generalized inverses*, Arch. Rational. Mech. Anal. **61** (1976), 197–251.
- [6] Q. LIU AND M. WEI, *Reverse order law for least squares  $g$ -inverses of multiple matrix products*, Linear and Multilinear Algebra **56** (2008), 491–506.
- [7] G. MARSAGLIA AND G. P. H. STYAN, *Equalities and inequalities for ranks of matrices*, Linear and Multilinear Algebra **2** (1974), 269–292.
- [8] R. PENROSE, *A generalized inverse for matrices*, Proc. Cambridge. Philos. Soc. **51** (1955), 406–413.
- [9] C. R. RAO AND S. K. MITRA, *Generalized Inverse of Matrices and Its Applications*, Wiley, New York, 1971.
- [10] W. SUN AND Y. WEI, *Inverse order rule for weighted generalized inverse*, SIAM J. Matrix Anal. Appl. **19** (1998), 772–775.
- [11] Y. TIAN, *Reverse order laws for generalized inverse of multiple matrix products*, Linear Algebra Appl. **211** (1994), 85–100.
- [12] G. WANG, Y. WEI AND S. QIAO, *Generalized Inverses: Theory and Computations*, Science Press, Beijing, 2004.
- [13] G. WANG AND B. ZHENG, *The reverse order law for the generalized inverse  $A_{T,S}^{(2)}$* , Appl. Math. Comput. **157** (2004), 295–305.
- [14] M. WEI, *Reverse order laws for generalized inverse of multiple matrix products*, Linear Algebra Appl. **293** (1999), 273–288.
- [15] M. WANG, M. WEI AND Z. JIA, *Mixed-type reverse order law of  $(AB)^{(1,3)}$* , Linear Algebra Appl. **43** (2009), 1691–1699.

- [16] Z. XIONG AND B. ZHENG, *The reverse order laws for  $\{1, 2, 3\}$ - and  $\{1, 2, 4\}$ -inverses of two matrix product*, Appl. Math. Let. **21** (2008), 649–655.
- [17] B. ZHENG AND Z. XIONG, *An new equivalent condition of the reverse order law for  $g$ -inverses of multiple matrix products*, Electronic Journal of Linear Algebra **17** (2008), 1–8.
- [18] B. ZHENG AND Z. XIONG, *On reverse order laws for the weighted generalized inverse*, Arab. J. Sci. Eng. **34** (2A) (2009), 195–203.
- [19] S. ZLOBEC, *An explicit form of the Moore-Penrose inverse of an arbitrary complex matrix*, SIAM Review **12** (1970), 132–134.