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SOME REVERSES OF THE CAUCHY—SCHWARZ INEQUALITY FOR COMPLEX FUNCTIONS OF SELF–ADJOINT OPERATORS IN HILBERT SPACES

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Abstract. We give some ratio and difference reverses of the Cauchy–Schwarz inequality for complex functions of self-adjoint operators in Hilbert spaces, under suitable assumptions for the involved operators. Several examples for particular functions of interest are provided as well.

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