COMMUTATORS FOR THE MAXIMAL FUNCTIONS
ON LEBESGUE SPACES WITH VARIABLE EXPONENT

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Abstract. Let M be the Hardy-Littlewood maximal function, the commutator generated by M and a suitable function b is defined by $[M,b]f = M(bf) - bMf$. In this paper, the authors give some characterizations of b for which $[M,b]$ is bounded on the Lebesgue spaces with variable exponent. The similar results are also proved for the commutator of the sharp maximal function.

Mathematics subject classification (2010): 42B25, 46E30.
Keywords and phrases: commutator, BMO, Hardy-Littlewood maximal function, sharp maximal function, Lebesgue space with variable exponent.

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