SOBOLEV EMBEDDINGS FOR RIESZ POTENTIALS
OF FUNCTIONS IN MORREY SPACES $L^{1,\nu,\beta_1,\beta_2}(\mathbf{R}^n)$

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Abstract. Our aim in this paper is to deal with Sobolev embeddings for Riesz potentials of functions in Morrey spaces $L^{1,\nu,\beta_1,\beta_2}(\mathbf{R}^n)$, as an extension of Trudinger [17], Serrin [14] and the authors [5] in the case of bounded open sets. We are mainly concerned with Trudinger's type exponential integrability.

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