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WIENER TYPE THEOREMS FOR FOURIER–VILENIKIN SERIES WITH NONNEGATIVE COEFFICIENTS AND SOLID SPACES

S. S. VOLOSIVETS

Abstract. In the present paper we show that L^p-integrability near zero of a function on Vilenkin group G with nonnegative Fourier-Vilenkin coefficients implies L^p-integrability on G, if p is even integer. This is an analog of N. Wiener-S. Wainger result. A refinement of Hausdorff-Young-F. Riesz inequality is obtained and several examples concerning embeddings of solid function spaces on G are given.

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