LIOUVILLE TYPE THEOREM FOR HIGHER ORDER HARDY–HÉNON SYSTEM OF INEQUALITIES

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Abstract. In this paper, we prove some new Liouville type theorems for fourth order and more general higher order Hardy-Hénon systems of inequalities. The test function method is applied to show the nonexistence of nontrivial nonnegative global solutions, which improve and extend some recent results.

REFERENCES

[1] S. N. AMSTRONG AND B. SIRAKOV, *Nonexistence of positive supersolutions of elliptic equations via the maximum principle*, Comm. Partial Differential Equations, **36**, 11 (2011), 2011–2047.

[2] M. F. BIDAUT-VÉRON AND H. GIACOMINI, *A new dynamical approach of Emden-fowler equations and systems*, Adv. Differential Equations, **15**, 11–12 (2010), 1033–1082.

[3] M. CALANCHI AND B. RUF, *Radial and non radial solutions for Hardy-Hénon type elliptic systems*, Calc. Var., **38**, (2010), 111–133.

[4] C. COWAN, *A Liouville theorem for a fourth order Hénon equation*, `http://arxiv.org/pdf/1110.2246.pdf`, (2011).

[5] L. DUPAIGNE AND A. C. PONCE, *Singularities of positive supersolutions in elliptic PDEs, Selecta Math. (N.S.)*, **10**, 3 (2004), 341–358.

[6] Y. V. EGOROV, V. A. GALAKTIONOV, V. A. KONDRATIEV AND S. POHOZAEV, *On the necessary conditions of global existence to a quasilinear inequality in the half-space*, C. R. Acad. Sci. Paris Sér. I. Math., **330**, (2000), 93–98.

[7] M. ESCOBEDO AND M. A. HERRERO, *Boundedness and blow up for a semilinear reaction-diffusion system*, J. Differential Equations, **89**, 1 (1991), 176–202.

[8] M. FAZLY AND N. GHOUSSOUB, *On the Hénon-Lane-Emden conjecture*, `http://arxiv.org/pdf/1107.5611.pdf`, (2011).

[9] J. FÖLDES, *Liouville theorems, a priori estimates, and blow-up rates for solutions of indefinite superlinear parabolic problems*, Czechoslovak Math. J., **61**, 136 (2011), 169–198.

[10] V. A. GALAKTIONOV AND S. I. POHOZAEV, *Existence and blow-up for higher-order semilinear parabolic equations: Majorizing order-preserving operators*, Indiana Univ. Math. J., **51**, 6 (2002), 1321–1338.

[11] B. GIDAS AND J. SPRUCK, *Global and local behavior of positive solutions of nonlinear elliptic equations*, Comm. Pure Appl. Math., **34**, 4 (1981), 525–598.

[12] Z. X. JIANG AND S. N. ZHENG, *Fujita-Liouville type theorem for coupled fourth-order parabolic inequalities*, Chin. Ann. Math., **33B**, 1 (2012), 107–112.

[13] A. G. KARTSATOS AND V. V. KURTA, *On a Liouville-type theorem and the Fujita blow-up phenomenon*, Proc. Amer. Math. Soc., **132**, (2004), 807–813.

[14] L. H. MIN, X. P. YANG AND C. F. GUI, *Entropy estimates and large time behavior of solutions to a fourth order nonlinear degenerate equation*, Commun. Contemp. Math., (2013), DOI: 10.1142/S0219199712500666.

[15] L. H. MIN AND X. P. YANG, *Finite speed of propagation and algebraic time decay of solutions to a generalized thin film equation*, Submitted to Communications on Pure and Applied Analysis (CPAA).

[16] E. MITIDIERI, *Nonexistence of positive solutions of semilinear elliptic systems in* $\mathbb{R}^N$, Differential Integral Equations, **9**, 3 (1996), 465–479.

[17] E. MITIDIERI AND S. I. POHOZAEV, *A priori estimates and the absence of solutions of nonlinear partial differential equations and inequalities*, Tr. Mat. Inst. Steklova, **234**, (2001), 1–384.

[18] Q. H. PHAN, *Liouville-type theorems and bounds of solutions for Hardy-Hénon elliptic systems*, Adv. Differential Equations, **17**, 7–8 (2012), 605–634.

[19] Q. H. PHAN AND PH. SOUPLET, *Liouville-type theorems and bounds of solutions of Hardy-Hénon equations*, J. Differential Equations, **252**, (2012), 2544–2562.

[20] P. POLÁČIK, P. QUITTNER AND PH. SOUPLET, *Singularity and decay estimates in superlinear problems via Liouville-type theorems, Part I: Elliptic equations and systems*, Duke Math. J., **139**, (2007), 555–579.

[21] P. POLÁČIK, P. QUITTNER AND PH. SOUPLET, *Singularity and decay estimates in superlinear problems via Liouville-type theorems, Part II: Parabolic equations*, Indiana Univ. Math. J., **56**, (2007), 879–908.

[22] PH. SOUPLET, *The proof of the Lane-Emden conjecture in four space dimensions*, Adv. Math., **221**, (2009), 1409–1427.

[23] X. W. XU, *Uniqueness theorem for the entire positive solutions of biharmonic equations in* $\mathbb{R}^n$, Proc. Roy. Soc. Edinburgh Sect. A, **130**, 3 (2000), 651–670.

[24] X. D. YAN, *A Liouville-type theorem for higher order elliptic systems*, J. Math. Anal. Appl., **387**, (2012), 153–165.

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