

MONOTONE MAPS ON DIAGONALIZABLE MATRICES

MIKHAIL A. EFIMOV AND ALEXANDER E. GUTERMAN

Abstract. We characterize injective maps preserving \leq^{\sharp} -order and maps strongly preserving \leq^{\sharp} -order on the set of diagonalizable matrices.

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REFERENCES

- [1] A. ALIEVA, A. GUTERMAN, *Monotone linear transformations on matrices are invertible*, Comm. in Algebra, **33** (2005), 3335–3352.
- [2] A. BEN-ISRAEL, T. GREVILLE, *Generalized Inverses: Theory and Applications*, New York: John Wiley and Sons, 1974.
- [3] I. I. BOGDANOV, A. E. GUTERMAN, *Monotone matrix maps defined by the group inverse and simultaneous diagonalizability*, Matematicheskii Sbornik, **198**, 1 (2007), 3–20 [in Russian].
- [4] J. DE PILLIS, *Linear transformations which preserve Hermitian and positive semidefinite operators*, Pacific J. of Math., **23** (1967), 129–137.
- [5] G. DOLINAR, A. GUTERMAN, J. MAROVT, *Automorphisms of $K(H)$ with respect to the star partial order*, Operators and Matrices, **7**, 1 (2013), 225–239.
- [6] G. DOLINAR, J. MAROVT, *Star partial order on $B(H)$* , Linear Algebra Appl., **434** (2011), 319–326.
- [7] M. A. EFIMOV, *Additive matrix maps that are monotone with respect to the orders induced by group inverse*, Fundamentalnaya i Prikladnaya Matematika, **17**, 6 (2011/2012), 23–40 [in Russian], Translated by Journal of Mathematical Sciences (New-York), **193**, 5 (2013), 659–670.
- [8] M. A. EFIMOV, *Linear matrix transformations that are monotone with respect to the \leq^{\sharp} - or \leq^{cn} -order*, Fundamentalnaya i Prikladnaya Matematika, **13**, 4 (2007), 53–66 [in Russian]. Translated by Journal of Mathematical Sciences (New-York), **155**, 6 (1008), 830–838.
- [9] M. A. EFIMOV, A. E. GUTERMAN, *Monotone maps on index one matrices*, Zapiski POMI, **405** (2012), 67–96 [in Russian], Translated by Journal of Mathematical Sciences (New-York), **191**, 1 (2013), 36–51.
- [10] M. H. ENGLEFIELD, *The commuting inverses of a square matrix*, Proc. Cambridge Philos. Soc., **62** (1966), 667–671.
- [11] I. ERDELYI, *On the matrix equation $Ax = \lambda Bx$* , Journal of Math. Anal. and Appl., **17** (1967), 117–132.
- [12] C.-A. FAURE, *An elementary proof of the fundamental theorem of projective geometry*, Geom. Dedicata, **90** (2002) 145–151.
- [13] A. GUTERMAN, *Linear preservers for Drazin star partial order*, Comm. in Algebra, **29**, 9 (2001), 3905–3917.
- [14] A. GUTERMAN, *Linear preservers for matrix inequalities and partial orderings*, Linear Algebra Appl., **331**, 1–3 (2001) 75–87.
- [15] R. E. HARTWIG, *How to partially order regular elements*, Math. Japonica, **25**, 1 (1980), 1–13.
- [16] R. E. HARTWIG, S. K. MITRA, *Partial orders based on outer inverses*, Linear Algebra Appl., **176** (1982), 3–20.
- [17] P. LEGIŠA, *Automorphisms of M_n , partially ordered by rank subtractivity ordering*, Linear Algebra Appl., **389** (2004), 147–158.

- [18] P. LEGIŠA, *Automorphisms of M_n , partially ordered by the star order*, Linear and Mult. Algebra., **54**, 3 (2006), 157–188.
- [19] S. K. MITRA, *A new class of g -inverse of square matrices*, Sankhyā, Ser. A, **30** (1963), 323–330.
- [20] S. K. MITRA, *On group inverses and the sharp order*, Linear Algebra Appl., **92** (1987), 17–37.
- [21] S. K. MITRA, P. BHIMASANKARAM, S. B. MALIK, *Matrix partial orders, shorted operators and applications*, London: Word Scientific, 2010.
- [22] K. S. S. NAMBOORIPAD, *The natural partial order on a regular semigroup*, Proceedings of the Edinburgh Math. Soc., **23** (1980), 249–260.
- [23] P. G. OVCHINNIKOV, *Automorphisms of the poset of skew projections*, J. of Functional Analysis, **115** (1993), 184–189.
- [24] C. R. RAO, S. K. MITRA, *Generalized Inverse of Matrices and its Applications*, New York: Wiley, 1971.
- [25] P. ROBERT, *On the group-inverse of a linear transformation*, Journal of Math. Anal. and Appl., **22** (1968), 658–669.
- [26] P. ŠEMRL, *Non-linear commutativity preserving maps*, Acta Sci. Math. (Szeged), **71** (2005), 781–819.
- [27] P. ŠEMRL, *Order-preserving maps on the poset of idempotent matrices*, Acta Sci. Math. (Szeged), **69** (2003), 481–490.