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ISOPERIMETRIC INEQUALITIES FOR POSITIVE SOLUTION OF P–LAPLACIAN

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Abstract. In this paper, we prove some isoperimetric inequalities and give a explicit bound for the positive solution of P-Laplacian.

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