

REFINEMENTS OF THE SHAFER–FINK INEQUALITY OF ARBITRARY UNIFORM PRECISION

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Abstract. A method of producing refinements of the Shafer–Fink ([5]) inequality

$$\frac{3x}{1+2\sqrt{1+x^2}} \leq \arctan x \leq \frac{\pi x}{1+2\sqrt{1+x^2}}$$

is given. We prove, for instance:

$$\frac{\pi(3+8\sqrt{2})x}{7+6\sqrt{1+x^2}+16\sqrt{2}\sqrt{1+x^2}+\sqrt{1+x^2}} \leq \arctan x \leq \frac{45x}{7+6\sqrt{1+x^2}+16\sqrt{2}\sqrt{1+x^2}+\sqrt{1+x^2}}.$$

Other algebraic approximations for the arctangent functions are, rather informally, presented.

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