VOLUME INEQUALITIES FOR ORLICZ MEAN ZONOIDS

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Abstract. In this paper, a more general mean zonoid called Orlicz mean zonoid $\overline{Z}_\phi K$ of a convex body K is introduced. Using shadow systems of convex bodies, we give a sharp lower estimate for the volume ratio of $\overline{Z}_\phi K$ and K, and a sharp upper estimate for the volume product of $\overline{Z}_\phi^* K$ and K.

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