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MORE JORDAN TYPE INEQUALITIES

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Abstract. The function $\tan(\pi x/2)/(\pi x/2)$ is expanded into a Laurent series of $1-x^2$, where the coefficients are given explicitly as combinations of zeta function of even integers. This is used to achieve a sequence of upper and lower bounds which are very precise even at the poles at $x=\pm 1$.

Similar results are obtained for other trigonometric functions with poles.

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