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ON AN INEQUALITY FOR THE RATIO OF GAMMA FUNCTIONS

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Abstract. The following inequality relating to the ratio of the gamma functions

$$\alpha \log x \leqslant x - \frac{\Gamma(x)}{\Gamma\left(x+\frac{1}{x}\right)},$$

where α is a suitable constant, is established for every $x > 0$. This inequality gives a contribution to the recent results, proved by several authors, involving the functions $\Gamma(x)$ and $\Gamma\left(\frac{1}{x}\right)$. It also gives an alternative proof of a conjecture formulated by D. Kershaw and recently proved by G.J.O. Jameson and T.P. Jameson [5].

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