

ON THE SUPER-STABILITY OF TRIGONOMETRIC HILBERT-VALUED FUNCTIONAL EQUATIONS

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Abstract. We generalize the super-stability result for cosine functional equation mappings with values in the field of complex numbers to the case of an arbitrary Hilbert space with the Hadamard product. On the other hand, by an example we prove that this generalization is not true for sine functional equation.

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