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LOCAL GRADIENT ESTIMATES FOR THE $p(x)$–LAPLACIAN ELLIPTIC EQUATIONS

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Abstract. In this paper we give a new and direct proof of local L^q estimates for the non-homogeneous $p(x)$-Laplacian elliptic equation under some proper conditions on $p(x) > 1$. We prove that

$$|\mathbf{f}|^{p(x)} \in L^q_{loc} \Longrightarrow |\nabla u|^{p(x)} \in L^q_{loc} \quad \text{for any } q \geqslant 1$$

for weak solutions of

$$\operatorname{div}\left(|\nabla u|^{p(x)-2}\nabla u\right) = \operatorname{div}\left(|\mathbf{f}|^{p(x)-2}\mathbf{f}\right) \quad \text{in } \Omega.$$

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