

## NEW BOUNDS FOR THE SPREAD OF THE SIGNLESS LAPLACIAN SPECTRUM

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*Abstract.* The spread of the signless Laplacian of a simple graph  $G$  is defined as  $SQ(G) = \mu_1(G) - \mu_n(G)$ , where  $\mu_1(G)$  and  $\mu_n(G)$  are the maximum and minimum eigenvalues of the signless Laplacian matrix of  $G$ , respectively. In this paper, we will present some new lower and upper bounds for  $SQ(G)$  in terms of clique and independence numbers. In the final section, as an application of the theory obtained in here, we will also show some new upper bounds for the spread of the signless Laplacian of tensor products of any two simple graphs.

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