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DOMINANCE OF ORDINAL SUMS OF THE ŁUKASIEWICZ AND THE PRODUCT TRIANGULAR NORM

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Abstract. In this paper we provide a simple characterization of the dominance in two classes of continuous triangular norms. In particular, we solve the dominance of (i) ordinal sum t-norms that use the Łukasiewicz t-norm as the only summand operation and (ii) ordinal sum t-norms that use the product t-norm as the only summand operation. In both cases, the dominance relation is characterized by a simple property of the idempotent elements of the dominating t-norm. We also introduce the notion of the axis of a conjunctor and, as a side result, we characterize dominance of continuous conjunctors in terms of their axes.

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