

## A MORE ACCURATE HALF-DISCRETE HILBERT-TYPE INEQUALITY WITH A GENERAL NON-HOMOGENEOUS KERNEL AND OPERATOR EXPRESSIONS

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*Abstract.* In this paper, by the use of the methods of weight functions and technique of real analysis, a more accurate half-discrete Hilbert-type inequality with a general non-homogeneous kernel and a best possible constant factor is given. The equivalent forms and some reverses are obtained. We also consider the operator expressions with the norm and some particular examples.

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### REFERENCES

- [1] G. H. HARDY, J. E. LITTLEWOOD, G. PÓLYA, *Inequalities*, Cambridge University Press, Cambridge, 1934.
- [2] D. S. MITRINOVIĆ, J. E. PEČARIĆ, A. M. FINK, *Inequalities involving functions and their integrals and derivatives*, Kluwer Academic Publishers, Boston, 1991.
- [3] B. C. YANG, *Hilbert-type integral inequalities*, Bentham Science Publishers Ltd., Sharjah, 2009.
- [4] B. C. YANG, *Discrete Hilbert-type inequalities*, Bentham Science Publishers Ltd., Sharjah, 2011.
- [5] B. C. YANG, *The norm of operator and Hilbert-type inequalities*, Science Press, Beijing, China, 2009.
- [6] B. C. YANG, *Hilbert-type integral operators: norms and inequalities*, Nonlinear analysis, stability, approximation, and inequalities (eds. P. M. Paralos et al.), Springer, New York, (2012), 771–859.
- [7] B. C. YANG, *On Hilbert's integral inequality*, Journal of Mathematical Analysis and Applications, **220**, (1998), 778–785.
- [8] B. C. YANG, I. BRNETIĆ, M. KRNIĆ, J. E. PEČARIĆ, *Generalization of Hilbert and Hardy-Hilbert integral inequalities*, Math. Ineq. and Appl., **8**, 2 (2005), 259–272.
- [9] M. KRNIĆ, J. E. PEČARIĆ, *Hilbert's inequalities and their reverses*, Publ. Math. Debrecen, **67** 3–4, (2005), 315–331.
- [10] B. C. YANG, TH. M. RASSIAS, *On the way of weight coefficient and research for Hilbert-type inequalities*, Math. Ineq. Appl., **6**, 4 (2003), 625–658.
- [11] B. C. YANG, TH. M. RASSIAS, *On a Hilbert-type integral inequality in the subinterval and its operator expression*, Banach J. Math. Anal., **4**, 2 (2010), 100–110.
- [12] L. AZAR, *On some extensions of Hardy-Hilbert's inequality and Applications*, Journal of Inequalities and Applications, 2009, no. 546829.
- [13] B. ARPAD, O. CHOONGHONG, *Best constant for certain multilinear integral operator*, Journal of Inequalities and Applications, 2006, no. 28582.
- [14] J. C. KUANG, L. DEBNATH, *On Hilbert's type inequalities on the weighted Orlicz spaces*, Pacific J. Appl. Math., **1**, 1 (2007), 95–103.
- [15] W. Y. ZHONG, *The Hilbert-type integral inequality with a homogeneous kernel of Lambda-degree*, Journal of Inequalities and Applications, 2008, no. 917392.
- [16] Y. HONG, *On Hardy-Hilbert integral inequalities with some parameters*, J. Ineq. in Pure & Applied Math., **6**, 4 (2005), Art. 92, 1–10.

- [17] W. Y. ZHONG, B. C. YANG, *On multiple Hardy-Hilbert's integral inequality with kernel*, Journal of Inequalities and Applications, Vol. 2007, Art.ID 27962, 17 pages, doi: 10.1155/2007/27.
- [18] B. C. YANG, M. KRNIĆ, *On the norm of a multi-dimensional Hilbert-type operator*, Sarajevo Journal of Mathematics, **7**, 20 (2011), 223–243.
- [19] Y. J. LI, B. HE, *On inequalities of Hilbert's type*. Bulletin of the Australian Mathematical Society, **76**, 1 (2007), 1–13.
- [20] B. C. YANG, *A mixed Hilbert-type inequality with a best constant factor*, International Journal of Pure and Applied Mathematics, **20**, 3 (2005), 319–328.
- [21] B. C. YANG, *A half-discrete Hilbert-type inequality*, Journal of Guangdong University of Education, **31**, 3 (2011), 1–7.
- [22] W. Y. ZHONG, *A mixed Hilbert-type inequality and its equivalent forms*, Journal of Guangdong University of Education, **31**, 5 (2011), 18–22.
- [23] W. Y. ZHONG, *A half discrete Hilbert-type inequality and its equivalent forms*, Journal of Guangdong University of Education, **32**, 5 (2012), 8–12.
- [24] J. H. ZHONG, B. C. YANG, *On an extension of a more accurate Hilbert-type inequality*, Journal of Zhejiang University (Science Edition), **35**, 2 (2008), 121–124.
- [25] J. H. ZHONG, *Two classes of half-discrete reverse Hilbert-type inequalities with a non-homogeneous kernel*, Journal of Guangdong University of Education, **32**, 5 (2012), 11–20.
- [26] W. Y. ZHONG, B. C. YANG, *A best extension of Hilbert inequality involving several parameters*, Journal of Jinan University (Natural Science), **28**, 1 (2007), 20–23.
- [27] W. Y. ZHONG, B. C. YANG, *A reverse Hilbert's type integral inequality with some parameters and the equivalent forms*, Pure and Applied Mathematics, **24**, 2 (2008), 401–407.
- [28] M. Y. ZHONG, B. C. YANG, *On multiple Hardy-Hilbert's integral inequality with kernel*, Journal of Inequalities and Applications, Vol. 2007, Art.ID 27962, 17 pages, doi: 10.1155/2007/27.
- [29] B. C. YANG, Q. CHEN, *A half-discrete Hilbert-type inequality with a homogeneous kernel and an extension*, Journal of Inequalities and Applications, **124** (2011), doi:10.1186/1029-242X-2011-124.
- [30] B. C. YANG, *A half-discrete Hilbert-type inequality with a non-homogeneous kernel and two variables*, Mediterranean Journal of Mathematics, 2012, doi: 10.1007/s00009-012-0213-50 online first.
- [31] B. C. YANG, *Two types of multiple half-discrete Hilbert-type inequalities*, Lambert Academic Publishing, Berlin, 2012.
- [32] J. C. KUANG, *Applied inequalities*, Shangdong Science Technic Press, Jinan, China, 2004.
- [33] J. C. KUANG, *Introduction to real analysis*, Hunan Education Press, Chansha, China, 1996.
- [34] Y. L. PAN, H. T. WANG, F. T. WANG, *On complex functions*, Science Press, Beijing, China, 2006.