

NECESSARY AND SUFFICIENT CONDITIONS FOR THE BOUNDEDNESS OF THE MAXIMAL OPERATOR FROM LEBESGUE SPACES TO MORREY-TYPE SPACES

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Abstract. It is proved that the boundedness of the maximal operator M from a Lebesgue space $L_{p_1}(\mathbb{R}^n)$ to a general local Morrey-type space $LM_{p_2\theta,w}(\mathbb{R}^n)$ is equivalent to the boundedness of the embedding operator from $L_{p_1}(\mathbb{R}^n)$ to $LM_{p_2\theta,w}(\mathbb{R}^n)$ and in its turn to the boundedness of the Hardy operator from $L_{p_1}(0,\infty)$ to the weighted Lebesgue space $L_{\frac{\theta}{p_2},\nu}(0,\infty)$ for a certain weight function ν determined by the functional parameter w . This allows obtaining necessary and sufficient conditions on the function w ensuring the boundedness of M from $L_{p_1}(\mathbb{R}^n)$ to $LM_{p_2\theta,w}(\mathbb{R}^n)$ for any $0 < \theta \leq \infty$, $0 < p_2 \leq p_1 \leq \infty$, $p_1 > 1$. These conditions with $p_1 = p_2 = 1$ are necessary and sufficient for the boundedness of M from $L_1(\mathbb{R}^n)$ to the weak local Morrey-type space $WLM_{1\theta,w}(\mathbb{R}^n)$.

Mathematics subject classification (2010): 42B20, 42B25, 42B35.

Keywords and phrases: Maximal operator, Morrey-type spaces, weak Morrey-type spaces.

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