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PERMUTATION INEQUALITIES

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Abstract. We formulate and solve some special cases of the following (in general NP-hard) extremal problem: "*Given a graph G (or a hypergraph H), label its vertices with given different n non-negative numbers $a_1 \geqslant a_2 \geqslant \ldots \geqslant a_n \geqslant 0$ in such a way that the sum of the products of labels in adjacent vertices $f = \sum a_i a_j$ will be maximal (or minimal)*". Solving this problem for some special families of graphs (e.g. paths, trees and stars) we obtain examples of "permutation inequalities" $f_{min} \leqslant f \leqslant f_{max}$.

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