GEOMETRIC CONSTANTS AND CHARACTERIZATIONS OF INNER PRODUCT SPACES

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Abstract. Let $X$ be a real normed space, let $\Psi_2$ denote the set of all convex functions on $[0,1]$ such that $\max\{1-t,t\} \leq \psi(t) \leq 1$, and let $\Phi_2$ denote the set of all concave function on $[0,1]$ such that $\psi(0) = \psi(1) = 1$. For each $\psi \in \Phi_2 \cup \Psi_2$, it is shown that
\[ \|x\|^{-1} \|x+y\|^{-1} \leq C_{\psi} \|x-y\| \quad \text{for all nonzero vectors } x, y \in X, \]
where $C_{\psi} = 4 \max \psi(t)$. The case of $\psi = \psi_p$ ($p > 0$), defined as $\psi_p(t) = ((1-t)p + tp)^{1/p}$, is due to Al-Rashed, and is due to Dunkl and Williams when $p = 1$. In particular, it is shown that for certain $\psi \in \Phi_2$, the inequality holds for $C_{\psi} = 2\psi(1/2)$ if and only if $X$ is an inner product space; this generalizes the works of Al-Rashed and Kirk-Smiley.


Keywords and phrases: inner product space, absolute normalized norm, BJ-orthogonality.

REFERENCES