ASYMPTOTIC EXPANSIONS OF THE LOGARITHM OF THE GAMMA FUNCTION IN THE TERMS OF THE POLYGAMMA FUNCTIONS

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Abstract. We present new asymptotic expansions of the logarithm of the gamma function in terms of the polygamma functions. Based on these expansions, we prove new complete monotonicity properties of some functions involving the gamma and polygamma functions. As consequences of them we establish new upper and lower bounds for the gamma function in terms of the polygamma functions.

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