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REVERSES OF THE TRIANGLE INEQUALITY IN INNER PRODUCT SPACES

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Abstract. We show that if $x_1,\cdots,x_n$ are vectors in a normed linear space $(X,\|\cdot\|)$ and $s_1,\cdots,s_n$ belong to the interval $[0,\infty)$, then

$$f_n(s_1,\cdots,s_n)=\sum_{j=1}^{n}\|s_jx_j\|-\left\|\sum_{j=1}^{n}s_jx_j\right\|$$

is a non-negative valued continuous function such that $f_n(s_1,\cdots,s_n)\leqslant f_n(t_1,\cdots,t_n)$ for all $s_1,\cdots,s_n$ and $t_1,\cdots,t_n$ in $[0,\infty)$ with $s_j\leqslant t_j$ $(1\leqslant j\leqslant n)$. By using it, we prove several versions of reverse triangle inequality in inner product spaces and discuss equality attainedness of norm inequalities in strictly convex Banach spaces.

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