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ON THE OHLIN LEMMA FOR HERMITE–HADAMARD–FEJÉR TYPE INEQUALITIES

TERESA RAJBA

Abstract. Using Ohlin's Lemma [21] on convex stochastic ordering, we get a simple proof of known Hermite-Hadamard-Fejér type inequalities. We also prove new inequalities. Using s-convex stochastic ordering [12], we also give some Hermite-Hadamard-Fejér type inequalities in the case of higher order convex functions. The obtained results are useful in proving some inequalities between the quadrature operators [31], [32].

Mathematics subject classification (2010): 26A51, 26D15, 60E15, 41A55.
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