

WEIGHTED LIPSCHITZ ESTIMATES FOR COMMUTATORS OF ONE-SIDED OPERATORS ON ONE-SIDED TRIEBEL–LIZORKIN SPACES

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Abstract. Using the extrapolation of one-sided weights, we establish the boundedness of commutators generated by weighted Lipschitz functions and one-sided singular integral operators from weighted Lebesgue spaces to weighted one-sided Triebel-Lizorkin spaces. The corresponding results for commutators of one-sided discrete square functions are also obtained.

Mathematics subject classification (2010): 42B20, 42B25.

Keywords and phrases: one-sided weight, weighted Lipschitz function, one-sided Triebel-Lizorkin space, one-sided singular integral, one-sided discrete square function.

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