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ON $(k,h;m)$–CONVEX MAPPINGS AND APPLICATIONS

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Abstract. In this paper, for given positive integer m and real functions k and h, we prove $(k,h;m)$-convexity of the mapping $\mathbf{p} \to \phi(\mathbf{p}) f\left(\frac{\Phi(\mathbf{p})}{\phi(\mathbf{p})}\right)$ with a convex (increasing) function f and a $(k,h;m)$-convex mapping Φ and a positive $(k,h;m)$-concave mapping ϕ. As application, we establish a subadditivity result for completely monotone and Bernstein functions. We also show monotonity of the mappings $\mathbf{p} \to \frac{\Phi(\mathbf{p})}{\phi(\mathbf{p})}$ and $\mathbf{p} \to f\left(\frac{\Phi(\mathbf{p})}{\phi(\mathbf{p})}\right)$ with respect to a group majorization combined with other preorders.

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