DIGITAL SUM INEQUALITIES AND APPROXIMATE
CONVEXITY OF TAKAGI–TYPE FUNCTIONS

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Abstract. For an integer $b \geqslant 2$, let $s_b(n)$ be the sum of the digits of the integer n when written in base b, and let $S_b(N) = \sum_{n=0}^{N-1} s_b(n)$. Several inequalities are derived for $S_b(N)$. Some of the inequalities can be interpreted as comparing the average value of $s_b(n)$ over integer intervals of certain lengths to the average value of a beginning subinterval. Two of the main results are applied to derive a pair of "approximate convexity" inequalities for a sequence of Takagi-like functions. One of these inequalities was discovered recently via a different method by V. Lev; the other is new.

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