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EIGENVALUE DECAY OF INTEGRAL OPERATORS GENERATED BY POWER SERIES–LIKE KERNELS

D. AZEVEDO AND V. A. MENEGATTO

Abstract. We deduce decay rates for eigenvalues of integral operators generated by power series-like kernels on a subset X of either $\mathbb{R}^q$ or $\mathbb{C}^q$. A power series-like kernel is a Mercer kernel having a series expansion based on an orthogonal family $\{f_\alpha\}_{\alpha\in\mathbb{Z}_+^q}$ in $L^2(X,\mu)$, in which μ is a complete measure on X. As so, we show that the eigenvalues of the integral operators are given by an explicit formula defined by the coefficients in the series expansion of the kernel and the elements of the orthogonal family. The inequalities and, in particular, the decay rates for the sequence of eigenvalues are obtained from decay assumptions on the sequence of coefficients in the expansion of the kernel and on the sequence $\{\|f_\alpha\|\}_{\alpha\in\mathbb{Z}_+^q}$.

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