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APPLICATIONS OF HÖLDER'S AND JENSEN'S INEQUALITIES IN STUDYING THE β–ABSOLUTE CONVERGENCE OF VILENKIN–FOURIER SERIES

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Abstract. In this paper the β-absolute convergence $(0 < \beta \leqslant 2)$ of Vilenkin-Fourier series for the functions of various classes of functions of generalized bounded fluctuation is studied. In proving our main results we use famous Hölder's inequality and Jensen's inequality for integrals. As a particular case our results give bounded Vilenkin group analogue of the corresponding circle group results of Schramm and Waterman [Acta. Math. Acad. Sci. Hungar 40 (3–4) (1982), 273–276]. One of our results generalizes the earlier result of Uno [Sci. Rep. Kanazawa Univ. 29 (2) (1984), 97–102]. It also generalizes the results of Onneweer [Duke Math. J. 39 (4) (1972), 599–609; Corollary 3 and Corollary 4].

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