CHARACTERIZATIONS FOR THE GENERALIZED
FRACTIONAL INTEGRAL OPERATORS ON MORREY SPACES

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Abstract. We present some characterizations for the boundedness of the generalized fractional integral operators on Morrey spaces. The characterizations follow from two key estimates, one for the norm of some functions in Morrey spaces, and another for the values of the corresponding fractional integrals. We prove three theorems about necessary and sufficient conditions. We show that these theorems are independent by giving some examples. We also obtain counterparts for the weak generalized Morrey spaces.

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