

COMPLETE MONOTONICITY PROPERTIES OF FUNCTIONS INVOLVING q -GAMMA AND q -DIGAMMA FUNCTIONS

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Abstract. In this paper, the q -analogue of the Stirling formula (the Moak formula) for the q -gamma function is exploited to prove the complete monotonicity property of functions involving the q -gamma and the q -digamma functions. The monotonicity of these functions is used to establish sharp inequalities for the q -gamma and the q -polygamma functions and the q -Harmonic number.

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