

## WEAK FORMS OF SUPERCYCLICITY AND A CLASS OF PARANORMAL OPERATORS

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*Abstract.* In this paper, we give several properties of class  $A$  operators, an interesting subclass of paranormal operators. In particular, we consider the operators  $T \in \mathcal{L}(\mathcal{H})$  such that  $T - \lambda$  is a class  $A$  operator for every  $\lambda \in \mathbb{C}$ . We also provide some cases for class  $A$  operators to have a nontrivial invariant subspace. Finally, we prove that there are no  $N$ -supercyclic class  $A$  operators with trivial kernel for any positive integer  $N$  and that weakly supercyclic class  $A$  operators with trivial kernel must be normal.

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