

BOUNDS FOR THE RATIOS OF DIFFERENCES OF POWER MEANS IN TWO ARGUMENTS

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Abstract. Using methods from classical analysis, sharp bounds for the ratio of differences of power means are obtained. Our results generalize and extend previous ones due to S. Wu (2005), and to S. Wu and L. Debnath (2011).

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