Mathematical
Inequalities
& Applications
Volume 17, Number 3 (2014), 989–1003 doi:10.7153/mia-17-72

FUNCTIONAL INEQUALITIES FOR THE BICKLEY FUNCTION

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Abstract. In this paper our aim is to deduce some complete monotonicity properties and functional inequalities for the Bickley function. The key tools in our proofs are the classical integral inequalities, like Chebyshev, Hölder-Rogers, Cauchy-Schwarz, Carlson and Grüss inequalities, as well as the monotone form of l'Hospital's rule. Moreover, we prove the complete monotonicity of a determinant function of which entries involve the Bickley function.

Mathematics subject classification (2010): 26A86, 39B62, 39B72.

Keywords and phrases: Bickley function, Turán type inequality, complete monotonicity, integral inequality, exponential convexity, log-convexity, geometrically concave function, Turán determinant, modified Bessel function of the second kind.

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