Mathematical
Inequalities
& Applications
Volume 17, Number 3 (2014), 1015–1026
doi:10.7153/mia-17-74

# THE SHARP INEQUALITIES RELATED TO WILKER TYPE

ZHEN-HANG YANG

*Abstract.* Wu and Srivastava have shown that for $t \in (0, \pi/2)$ the inequalities

$$\left(\frac{\sqrt{\cos^{2q} t + 8} + \cos^q t}{4}\right)^{1/q} < \frac{\sin t}{t} < \left(\frac{\sqrt{\cos^{2p} t + 8} + \cos^p t}{4}\right)^{1/p}$$

hold if $q < 0$ and $p \geqslant 1$. In this paper we find the largest $q = 3/5$ and the smallest $p = \frac{\ln 2}{2(\ln \pi - \ln 2)}$ such that these inequalities hold. Moreover, our results also imply a type of new inequalities for trigonometric functions and give an anwser for a problem posed by Zhu.

*Mathematics subject classification* (2010): 26D05, 26D07, 33B10.
*Keywords and phrases*: Sharp bound, trigonometric function, Wilker type inequality.

## REFERENCES

[1] A. BARICZ AND J. SÁNDOR, *Extensions of generalized Wilker inequality to Bessel functions*, J. Math. Inequal. **2** (2008), 397–406.
[2] C.-P. CHEN AND W.-S. CHEUNG, *Wilker- and Huygens-type inequalities and solution to Oppenheim's problem*, Integral Transforms Spec. Funct. **23**, 5 (2012), 325–336.
[3] C.-P. CHEN, W.-S. CHEUNG, *Sharpness of Wilker and Huygens type inequalities*, J. Inequal. Appl. **2012** (2012): 72.
[4] J.-C. KUANG, *Applied Inequalities*, Shangdong Science and Technology Press, Jinan, China, 3rd edition, 2004 (in Chinese).
[5] I. LAZAREVIĆ, *Neke nejednakosti sa hiperbolickim funkcijama*, Univerzitet u Beogradu, Publikacije Elektrotehničkog Fakulteta, Serija Matematika i Fizika **170** (1966), 41–48.
[6] D. S. MITRINOVIČ, *Analytic Inequalities*, Springer-Verlag, Berlin, 1970.
[7] C. MORTICI, *The natural approach of Wilker-Cusa-Huygens inequalities*, Math. Inequal. Appl. **14**, 3 (2011), 535–541.
[8] E. NEUMAN AND J. SÁNDOR, *On some inequalities involving trigonometric and hyperbolic functions with emphasis on the Cusa-Huygens, Wilker, and Huygens inequalities*, Math. Inequal. Appl. **13**, 4 (2010), 715–723.
[9] E. NEUMAN, *On Wilker and Huygens type inequalities*, Math. Inequal. Appl. **15**, 2 (2012), 271–279.
[10] J. SÁNDOR AND M. BENCZE, *On Huygens's trigonometric inequality*, RGMIA Research Report Collection **8**, 3 (2005); available online at `http://www.ajmaa.org/RGMIA/papers/v8n3/Huygens.pdf`.
[11] J. S. SUMNER, A. A. JAGERS, M. VOWE AND J. ANGLESIO, *Inequalities involving trigonometric functions*, Amer. Math. Monthly **98** (1991), 264–267.
[12] J. B. WILKER, *Problem E3306*, Amer. Math. Monthly **96**, 1 (1989), 55.
[13] S.-H. WU AND H. M. SRIVASTAVA, *A weighted and exponential generalization of Wilker's inequality and its applications*, Integral Transforms and Spec. Funct. **18** (2007), 529–535.
[14] S.-H. WU AND H. M. SRIVASTAVA, *A further refinement of Wilker's inequality*, Integral Transforms and Spec. Funct. **19** (2008), 757–765.
[15] S.-H. WU, *On extension and refinement of Wilker's inequality*, Rocky Mountain J. Math. **39** (2009), 683–687.


Paper MIA-17-74

[16] S.-H. WU AND L. DEBNATH, *Wilker-type inequalities for hyperbolic functions*, Appl. Math. Lett. **25**, 5 (2012), 837–842.
[17] ZH.-H. YANG, *Sharp bounds for Seiffert mean in terms of weighted power means of arithmetic mean and geometric mean*, Math. Inequal. Appl. in press.
[18] L. ZHANG AND L. ZHU, *A new elementary proof of Wilker's inequalities*, Math. Inequal. Appl. **11**, 1 (2008), 149–151.
[19] L. ZHU, *On Wilker-type inequalities*, Math. Inequal. Appl. **10**, 4 (2007), 727–731.
[20] L. ZHU, *Some new Wilker-type inequalities for circular and hyperbolic functions*, Abstr. Appl. Anal. **2009** (2009), Art. ID 485842.
[21] L. ZHU, *A source of inequalities for circular functions*, Comput. Math. Appl. **58** (2009), 1998–2004.
[22] L. ZHU, *Inequalities for Hyperbolic functions and their Applications*, J. Inequal. Appl. **2010** (2010), Art. ID 130821.