

THE TRIANGLE INEQUALITY FOR GRADED REAL VECTOR SPACES OF LENGTH 3 AND 4

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Abstract. In [3] the classical theorem of Minkowski on lattice points and convex bodies in \mathbb{R}^n was generalized to simply connected nilpotent Lie groups with a grading of length 2. In doing so it was necessary to prove the triangle inequality for a certain natural homogeneous norm (with respect to automorphisms) of the Lie algebra associated with the grading (the case of a grading of length 1 being the Schwarz inequality). Here we shall extend the homogeneous norms for which the triangle inequality holds to gradings of length 3 and 4. The results hold for any graded real vector space of those lengths.

Mathematics subject classification (2010): 17B70, 22E25, 26D15.

Keywords and phrases: Graded Lie algebra, subadditive homogeneous norm.

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