

THE λ -INTERSECTION BODIES AND AN ANALYTIC GENERALIZED BUSEMANN-PETTY PROBLEM

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Abstract. In this paper, we obtain an extension of connections between an analytic generalization of the Busemann-Petty problem and positive definite distributions. We show that the class of the λ -intersection bodies is closely related to the analytic generalized Busemann-Petty problem.

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