ON A QUESTION FOR DUAL QUERMASSINTEGRALS

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Abstract. The well-known question for dual quermassintegrals is following: For which values of $i \in \mathbb{N}$ and every pair of star bodies K and L, is it true that

$$\frac{\tilde{W}_i(K \tilde{+} L)}{\tilde{W}_{i+1}(K \tilde{+} L)} \leqslant \frac{\tilde{W}_i(K)}{\tilde{W}_{i+1}(K)} + \frac{\tilde{W}_i(L)}{\tilde{W}_{i+1}(L)} \quad ?$$

In 2005, the inequality was proved if $i = n - 1$ or $i = n - 2$. But, now the question is not solved completely for $0 \leqslant i \leqslant n - 3$ and every pair of star bodies K and L. In the paper, we will give a completely answer for the dual question.

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REFERENCES

[1] R. SCHNEIDER, *Convex Bodies: The Brunn-Minkowski Theory*, Cambridge Univ. Press, 1993.

[2] Y. D. BURAGO, V. A. ZALGALLER, *Geometric Inequalities*, Springer-Verlag, Berlin-New york, 1988.

[3] E. LUTWAK, *Intersection bodies and dual mixed volumes*, Adv. Math. **71** (1988), 232–261.

[4] R. J. GARDNER, *Geometric Tomography*, Cambridge Univ. Press, New York, 1996.

[5] E. LUTWAK, *Dual mixed volumes*, Pacific J. Math. **58** (1975), 531–538.

[6] M. MARCUS, I. LOPES, *Inequalities for symmetric functions and Hermitian matrices*, Canad J. Math. **8** (1956), 524–531.

[7] E. F. BECHENBACH, R. BELLMAN, *Inequalities*, Springer-Verlag, Berlin-Göttingen, Heidelberg, 1961.

[8] H. BERGSTROM, *A triangle inequality for matrices. Den Elfte Skandinaviski Matematiker-kongress*, Trondheim, 1949, Oslo: John Grundt Tanums Forlag, 1952.

[9] R. BELLMAN, *Notes on matrix theory-IV: an inequality due to Bergstrom*, Amer. Math. Monthly **62** (1955), 172–173.

[10] K. FAN, *Some inequalities conerning positive-definite hermitian matrices*, Proc. Cambridge Phil. Soc. **51** (1955), 414–421.

[11] K. FAN, *Problem 4786*, Amer. Math. Monthly **65** (1958), 289.

[12] L. MIRSKY, *Maximum principles in matrix theory*, Proc. Glasgow Math. Assoc. **4** (1958), 34–37.

[13] M. FRADELIZI, A. GIANNOPOULOS AND M. MEYER, *Some inequalities about mixed volumes*, Israel J. Math. **135** (2003), 157–179.

[14] A. GIANNOPOULOS, M. HARTZOULAKI AND G. PAOURIS, *On a local version of the Aleksandrov-Fenchel inequality for the quermassintegrtals of a convex body*, Proc. Amer. Math. Soc. **130** (2002), 2403–2412.

[15] A. DEMBO, T. M. COVER AND J. A. THOMAS, *Information theoretic inequalities*, IEEE Trans. Infor. Theory **37** (1991), 1501–1518.

[16] X. Y. LI, G. S. LENG, *Some inequalities about dual mixed volumes of star bodies*, Acta Math. Sci. **25** (2005), 505–510.

[17] X. Y. LI, *Inequalities and extremum problems for convex bodies and star bodies*, The dissertation for the degree of doctor, 2004, 35–36.

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