# SOME NEW SCALES OF REFINED JENSEN
# AND HARDY TYPE INEQUALITIES

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*Abstract.* Some scales of refined Jensen and Hardy type inequalities are derived and discussed. The key object in our technique is $\gamma$-quasiconvex functions $K(x)$ defined by $K(x)x^{-\gamma} = \varphi(x)$, where $\varphi$ is convex on $[0, b)$, $0 < b \leqslant \infty$ and $\gamma \geqslant 0$.

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