

RIEMANN SUMS FOR SELF-ADJOINT OPERATORS

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Abstract. This paper focuses on Riemann sums for the functional calculus of bounded self-adjoint operators. We first obtain some monotonicity properties of operator convex functions. Using these results we then refine an operator Hermite–Hadamard type inequality. Finally we extend the Alzer and Bennet inequalities to operators on Hilbert spaces.

Mathematics subject classification (2010): 47A63, 15A42, 47A30.

Keywords and phrases: Hermite–Hadamard inequality, operator convex function, Alzer inequality, Bennet inequality, operator inequality.

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