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DETERMINATION OF ORDER OF MAGNITUDE OF MULTIPLE FOURIER COEFFICIENTS OF FUNCTIONS OF BOUNDED ϕ–VARIATION HAVING LACUNARY FOURIER SERIES USING JENSEN'S INEQUALITY

BHIKHA LILA GHODADRA

Abstract. For a Lebesgue integrable complex-valued function f defined over the m-dimensional torus $\mathbb{T}^m := [0,2\pi)^m$, let $\hat{f}(\mathbf{n})$ denote the Fourier coefficient of f, where $\mathbf{n} = (n^{(1)},\ldots,n^{(m)}) \in \mathbb{Z}^m$. Recently, in one of our papers [to appear in Mathematical Inequalities & Applications], we have defined the notion of bounded ϕ-variation for a complex-valued function on a rectangle $[a_1,b_1]\times\ldots\times[a_m,b_m]$ and studied the order of magnitude of Fourier coefficients of such functions on $[0,2\pi]^m$. In this paper, the order of magnitude of Fourier coefficients of a function of bounded ϕ-variation from $[0,2\pi]^m$ to $\mathbb{C}$ and having lacunary Fourier series with certain gaps is studied and a generalization of our earlier result (Theorem in [Acta Sci. Math. (Szeged), 78, (2012), 97–109]) is proved. Interestingly, the Jensen's inequality for integrals is used to prove the main result.

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