Mathematical
Inequalities
& Applications
Volume 17, Number 4 (2014), 1201–1223
doi:10.7153/mia-17-90

CONVOLUTION INEQUALITIES IN WEIGHTED LORENTZ SPACES

MARTIN KŘEPELA

Abstract. We characterize boundedness of a convolution operator with a fixed kernel between the weighted Lorentz spaces $\Lambda^p(v)$ and $\Gamma^q(w)$ for $0 < p \leqslant q \leqslant \infty$, $1 \leqslant q < p < \infty$ and $0 < q \leqslant p = \infty$. We provide corresponding weighted Young-type inequalities and also study basic properties of some new involved r.i. spaces.

Mathematics subject classification (2010): 44A35, 26D10, 46E30.
Keywords and phrases: Convolution, Young inequality, O'Neil inequality, Lorentz spaces, weights.

REFERENCES

[1] C. BENNETT, R. SHARPLEY, *Interpolation of operators*, Pure and Applied Mathematics, 129. Academic Press, Boston, 1988.
[2] A. P. BLOZINSKI, *Convolution of $L(p,q)$ functions*, Proc. Amer. Math. Soc., **(32)**, 237–240, 1972.
[3] M. CARRO, A. GOGATISHVILI, J. MARTÍN, L. PICK, *Weighted inequalities involving two Hardy operators with applications to embeddings of function spaces*, J. Operator Theory, **(59)**, 309–332, 2008.
[4] M. CARRO, L. PICK, J. SORIA, V. D. STEPANOV, *On embeddings between classical Lorentz spaces*, Math. Inequal. Appl. **(4)**, 397–428, 2001.
[5] M. CARRO, J. SORIA, *Boundedness of some integral operators*, Canad. J. Math., **(45)**, 1155–1166, 1993.
[6] A. CIANCHI, L. PICK, *Sobolev embeddings into spaces of Campanato, Morrey, and Hölder type*, J. Math. Anal. Appl. **(282)**, 128–150, 2003.
[7] M. ĆWIKEL, A. KAMIŃSKA, L. MALIGRANDA, L. PICK, *Are generalized Lorentz "spaces" really spaces?*, Proc. Amer. Math. Soc., **(132)**, 3615–3625, 2004.
[8] A. GOGATISHVILI, V. D. STEPANOV, *Reduction theorems for operators on the cones of monotone functions*, J. Math. Anal. Appl., **(405)**, 156–172, 2013.
[9] L. HÖRMANDER, *Estimates for translation invariant operators in L^p spaces*, Acta Math., **(104)**, 93–140, 1960.
[10] R. A. HUNT, *On $L(p,q)$ spaces*, Enseign. Math., **(12)**, 249–276, 1966.
[11] A. KAMIŃSKA, L. MALIGRANDA, *On Lorentz spaces $\Gamma_{p,w}$*, Israel J. Math., **(140)**, 285–318, 2004.
[12] A. KUFNER, L.-E. PERSSON, *Weighted inequalities of Hardy type*, World Scientific Publishing Co., River Edge, 2003.
[13] E. A. MYASNIKOV, L.-E. PERSSON, V. D. STEPANOV, *On the best constants in certain integral inequalities for monotone functions*, Acta Sci. Math. (Szeged), **(59)**, 613–624, 1994.
[14] E. NURSULTANOV, S. TIKHONOV, *Convolution inequalities in Lorentz spaces*, J. Fourier Anal. Appl., **(17)**, 486–505, 2011.
[15] R. OINAROV, *Two-sided estimates of the norm of some classes of integral operators*, Proc. Steklov Inst. Math., **(204)**, 205–214, 1994.
[16] R. O'NEIL, *Convolution operators and $L(p,q)$ spaces*, Duke Math. J., **(30)**, 129–142, 1963.
[17] E. SAWYER, *Boundedness of classical operators on classical Lorentz spaces*, Studia Math., **(96)**, 145–158, 1990.
[18] R. SHARPLEY, *Counterexamples for classical operators on Lorentz-Zygmund spaces*, Studia Math., **(68)**, 141–158, 1980.
[19] V. D. STEPANOV, *The weighted Hardy's inequality for nonincreasing functions*, Trans. Amer. Math. Soc., **(338)**, 173–186, 1993.

Paper MIA-17-90

[20] L. Y. H. YAP, *Some remarks on convolution operators and $L(p,q)$ spaces*, Duke Math. J., **(36)**, 647–658, 1969.