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CONCENTRATION–COMPACTNESS PRINCIPLE FOR GENERALIZED MOSER–TRUDINGER INEQUALITIES: CHARACTERIZATION OF THE NON–COMPACTNESS IN THE RADIAL CASE

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Abstract. Let $B(R) \subset \mathbb{R}^n$, $n \geqslant 2$, be an open ball. By a result from [1], the Moser functional with the borderline exponent from the Moser inequality fails to be sequentially weakly continuous on the set of radial functions from the unit ball in $W_0^{1,n}(B(R))$ only in the exceptional case of sequences acting like a concentrating Moser sequence (in particular, these sequences are weakly converging to zero).

We extend this result to the case of a nontrivial weak limit and the Moser functional with the borderline exponent from the Concentration-Compactness Alternative. The same result is obtained for the Orlicz-Sobolev space $W_0L^n \log^\alpha L(B(R))$ with $\alpha < n-1$. We also consider the case of Orlicz-Sobolev spaces embedded into multiple exponential spaces.

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