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UPPER BOUNDS FOR THE COVERING NUMBER OF CENTRALLY SYMMETRIC CONVEX BODIES IN $\mathbb{R}^n$

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Abstract. The covering number $c(K)$ of a convex body K is the least number of smaller homothetic copies of K needed to cover K. We provide new upper bounds for $c(K)$ when K is centrally symmetric by introducing and studying the generalized α-blocking number $\beta_2^{\alpha}(K)$ of K. It is shown that when a centrally symmetric convex body K is sufficiently close to a centrally symmetric convex body K', then $c(K)$ is bounded by $\beta_2^{\alpha}(K')$ from above, where α is a properly chosen number. Related results in Minkowski geometry are also presented.

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