

WEIGHTED ESTIMATES FOR VECTOR-VALUED COMMUTATORS OF GENERALIZED FRACTIONAL INTEGRALS

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Abstract. In this paper, the authors study the vector-valued commutator of generalized fractional integral operator $I_{\alpha,b,q}^k$ where the kernel K_α satisfies some conditions associated with the Young functions. The authors prove the two-weight norm inequalities for $I_{\alpha,b,q}^k$ where the weight ω is only a local integrable function. As an application of the main theorems in this paper, the weighted boundedness for vector-valued commutators of fractional integral with a rough kernel is also given.

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