

## STRONG APPROXIMATION OF SOME ADDITIVE FUNCTIONALS OF SYMMETRIC STABLE PROCESS

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*Abstract.* This paper deals with some additive functionals based on the local time of symmetric stable process. In concrete, we obtain some  $L_p$ -inequalities of the local time and the fractional derivative of the local time of symmetric stable process of index  $1 < \alpha \leq 2$ . As an application, we generalize the well known Barlow-Yor [4] inequality, which we use to give a strong approximation version, (almost surely estimate), of occupation times problem of this process. Our results generalize those obtained by Csaki et al. [7] for Brownian motion, and Ait Ouahra and Ouali [2] for symmetric stable process of index  $1 < \alpha \leq 2$  in  $L_p$ -norm.

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### REFERENCES

- [1] M. AIT OUAHRA AND M. EDDAHBI, *Théorèmes limites pour certaines fonctionnelles associées aux processus stable sur l'espace de Hölder*, Publ. Math. **45** (2) (2001), 371–386.
- [2] M. AIT OUAHRA AND M. OUALI, *Occupation time problems for fractional Brownian motion and some other self-similar processes*, Rand. Oper. Stoch. Equa. **17**, Issue 1 (2009), 69–89.
- [3] M. T. BARLOW, *Necessary and sufficient conditions for the continuity of local times of Lévy processes*, Ann. Probab. **16** (4) (1988), 1389–1427.
- [4] M. T. BARLOW AND M. YOR, *Semi-martingale inequalities via the Garcia-Rodemich-Rumsey lemma, and applications to local times*, J. Funct. Anal. **49** (1982), 198–229.
- [5] T. BOJDECKI, L. GOROSTIZA, A. TALARCZYK, *Sub-fractional Brownian motion and its relation to occupation times*, Statist. Probab. Lett. **69** (2004), 405–419.
- [6] E. S. BOYLAN, *Local times for a class of Markov processes*, Illinois J. Math. **8** (1964), 19–39.
- [7] E. CSAKI, Z. SHI AND M. YOR, *Fractional Brownian motions as “higher-order” fractional derivatives of Brownian local times*, in: Limit Theorems in Probability and Statistics, I. Balatonlelle, Janos Bolyai Math. Soc., Budapest (2000), 365–387.
- [8] E. CSAKI, M. CSÖRGŐ, A. FÖLDES AND Z. SHI, *Path properties of Cauchy's principal values related to local time*, Studia. Sci. Math. Hungarica. **38** (2001), 149–169.
- [9] P. J. FITZSIMMONS AND R. K. GETOOR, *Limit theorems and variation properties for fractional derivatives of the local time of a stable process*, Ann. Inst. H. Poincaré. **28** (2) (1992), 311–333.
- [10] M. B. MARCUS AND J. ROSEN,  *$p$ -variation of the local times of symmetric stable processes and of Gaussian processes with stationary increments*, Ann. Probab. **20** (4) (1992), 1685–1713.
- [11] S. G. SAMKO, A. A. KILBAS AND O. I. MARICHEV, *Fractional integrals and derivatives. Theory and applications*, Gordon and Breach Science Publishers, Yverdon (1993).