

INEQUALITIES AND ASYMPTOTIC EXPANSIONS RELATED TO GLAISHER–KINKELIN CONSTANT

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Abstract. We present sharp inequalities and asymptotic expansions related to Glaisher–Kinkelin constant. Also, we present sharp inequality and asymptotic expansion for sum $\sum_{k=1}^n k \ln \left(1 + \frac{1}{k}\right)$. This solves an open problem proposed in 2009 by Mihály Bencze.

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