

ON ONE EXTENSION THEOREM DEALING WITH WEIGHTED ORLICZ–SLOBODETSKII SPACE. ANALYSIS ON CUBE

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Abstract. Having given weight $\tilde{\rho} = \rho(\text{dist}(x, \partial Q))$ defined on cube Q and Orlicz function R , we construct the weight $\omega_\rho(\cdot, \cdot)$ defined on $\partial Q \times \partial Q$ and extension operator $\text{Ext}^L: \text{Lip}_d(\partial Q) \mapsto \text{Lip}(Q)$ from Lipschitz functions defined on ∂Q with certain restricted support to Lipschitz functions defined on Q , independent of ρ and R , in such a way that Ext^L extends to the bounded operator from certain subspace of weighted Orlicz-Slobodetskii space $Y_{\omega_\rho}^{R,R}(\partial Q)$ subordinated to the weight ω_ρ to Orlicz Sobolev space $W_\rho^{1,R}(Q)$. Result is new in the unweighted Orlicz setting for general function R as well as in the weighted L^p setting.

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