Mathematical
Inequalities
& Applications
Volume 18, Number 1 (2015), 97–110
doi:10.7153/mia-18-06

THE MAXIMAL OPERATOR OF MARCINKIEWICZ–FEJÉR MEANS WITH RESPECT TO WALSH–KACZMARZ–FOURIER SERIES

KÁROLY NAGY

Abstract. In the paper [4, Theorem 1] Gát, Goginava and the author proved that the maximal operator $\sigma^{\kappa,*}$ of Marcinkiewicz-Fejér means of Walsh-Kaczmarz-Fourier series, is bounded from the dyadic Hardy space H_p into the space L_p for $p > 2/3$. Moreover, Goginava and the author showed that $\sigma^{\kappa,*}$ is not bounded from the Hardy space $H_{2/3}$ to the space $L_{2/3}$ [6, Theorem 1]. The main aim of this paper is to show that the maximal operator $\tilde{\sigma}^{\kappa,*} f := \sup_{n\in\mathbb{P}} \frac{|\sigma_n^{\kappa} f|}{\log^{3/2}(n+1)}$, is bounded from the Hardy space $H_{2/3}$ into the space $L_{2/3}$. Moreover, we prove that the order of deviant behavior of the nth Walsh-Kacmarz-Marcinkiewicz-Fejér mean is exactly $\log^{3/2}(n+1)$ in the endpoint $p = 2/3$.

Mathematics subject classification (2010): 42C10.
Keywords and phrases: Walsh-Kaczmarz system, Marcinkiewicz means, maximal operator, two-dimensional system.

REFERENCES

[1] G. N. AGAJEV, N. YA. VILENKIN, G. M. DZHAFARLI AND A. I. RUBINSTEIN, *Multiplicative systems of functions and harmonic analysis on 0-dimensional groups*, "ELM" (Baku, USSR) (1981) (Russian).
[2] G. GÁT, *On* $(C,1)$ *summability of integrable functions with respect to the Walsh-Kaczmarz system*, Studia Math. **130 (2)** (1998), 135–148.
[3] G. GÁT, U. GOGINAVA AND K. NAGY, *On* (H_{pq},L_{pq})*-type inequality of maximal operator of Marcinkiewicz-Fejér means of double Fourier series with respect to the Walsh-Kaczmarz system*, Math. Ineq. Appl. **9 (3)** (2006), 473–485.
[4] G. GÁT, U. GOGINAVA AND K. NAGY, *On the Marcinkiewicz-Fejér means of double Fourier series with respect to the Walsh-Kaczmarz system*, Studia Sci. Math. Hungar. **46 (3)** (2009), 399–421.
[5] U. GOGINAVA, *The maximal operator of the Fejér means of the character system of the* p*-series field in the Kaczmarz rearrangement*, Publ. Math. Debrecen **71 (1-2)** (2007), 43–55.
[6] U. GOGINAVA AND K. NAGY, *On the maximal operator of the Marcinkiewicz-Fejér means of double Walsh-Kaczmarz-Fourier series*, Publ. Math. Debrecen **75 (1-2)** (2009), 95–104.
[7] U. GOGINAVA AND K. NAGY, *On the maximal operator of Walsh-Kaczmarz-Fejér means*, Czechoslovak Mathematical Journal **61 (136)** (2011), 673–686.
[8] J. MARCINKIEWICZ, *Sur une methode remarquable de sommation des series doubles de Fourier*, Ann. Scuola Norm. Sup. Pisa **8**, (1939), 149–160.
[9] K. NAGY, *Some convergence properties of the Walsh-Kaczmarz system with respect to the Marcinkiewicz means*, Rendiconti del Circolo Matematico di Palermo, Serie II, Suppl. 76 (2005), 503–516.
[10] K. NAGY, *On the maximal operator of Walsh-Marcinkiewicz means*, Publ. Math. Debrecen **78 (3-4)** (2011), 633–646.
[11] F. SCHIPP, W. R. WADE, P. SIMON, AND J. PÁL, *Walsh Series. An Introduction to Dyadic Harmonic Analysis*, Adam Hilger (Bristol-New York 1990).
[12] F. SCHIPP, *Pointwise convergence of expansions with respect to certain product systems*, Anal. Math. **2** (1976), 63–75.

Paper MIA-18-06

[13] P. SIMON, *On the Cesàro summability with respect to the Walsh-Kaczmarz system*, J. Approx. Theory **106** (2000), 249–261.
[14] V. A. SKVORTSOV, *On Fourier series with respect to the Walsh-Kaczmarz system*, Anal. Math. **7** (1981), 141–150.
[15] A. A. ŠNEIDER, *On series with respect to the Walsh functions with monotone coefficients*, Izv. Akad. Nauk SSSR Ser. Math. **12** (1948), 179–192.
[16] W. S. YOUNG, *On the a.e convergence of Walsh-Kaczmarz-Fourier series*, Proc. Amer. Math. Soc. **44** (1974), 353–358.
[17] F. WEISZ, *Martingale Hardy spaces and their applications in Fourier analysis*, Springer-Verlang, Berlin, 1994.
[18] F. WEISZ, *Summability of multi-dimensional Fourier series and Hardy space*, Kluwer Academic, Dordrecht, 2002.
[19] F. WEISZ, *θ-summability of Fourier series*, Acta Math. Hungar. **103** (2004), 139–176.
[20] F. WEISZ, *Convergence of double Walsh-Fourier series and Hardy spaces*, Approx. Theory Appl. **17** (2001), 32–44.
[21] L. V. ZHIZHIASHVILI, *Generalization of a theorem of Marcinkiewicz*, Izv. Akad. Nauk USSR Ser Math. **32** (1968), 1112–1122.